WHAT IS MEREOLOGICAL HARMONY?

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Forthcoming in Synthese

Abstract

Say that mereological harmony is the view that there is at least some mirroring between the mereological structure of material objects and the mereological structure of their locations: each, in some way, mirrors the other. As it turns out, there is a confusing array of systems of harmony available to the substantivalist. In this paper, I attempt to bring some order to these systems. I explore some systems found in the literature, as well as some natural systems which haven’t been discussed. Along the way, I explore a number of metaphysical consequences of the different systems of harmony. The paper ends with a roadmap of possible views for the substantivalist.

The material world has a certain mereological structure to it. And so does the spatiotemporal world. One very natural thought to have is that it is impossible for something \( x \) to be a part of \( y \) without \( x \)’s location also being a part of \( y \)’s location. For instance, it is impossible for my arm to be a part of my body without my arm’s location being a part of my body’s location. And a second very natural thought to have is that it is impossible for \( x \)’s location to be a part of \( y \)’s location without \( x \) also being a part of \( y \). Call cases where these misalignments do in fact occur parthood misalignment cases. The former thought suggests that the mereological structure of spacetime mirrors the mereological structure of the material world with respect to the parthood relation and the latter thought suggests that the mereological structure of material objects mirrors the mereological structure of spacetime with respect to the parthood relation. Very few people, I think, are willing to accept the possibility of the first kind of misalignment. The second type of parthood misalignment is a bit more controversial. However, I can imagine
a number of people wanting to accept the following thesis:

It is impossible for \( x \) to be a part of \( y \) without \( x \)’s location being a part of \( y \)’s location, and vice versa. \[ \text{No Parthood Misalignment (No PM)} \]

If one is tempted to accept this thesis then, presumably, one will be tempted by the thesis of mereological harmony. Say that mereological harmony is the view that there is, in both directions, at least \emph{some} mirroring between these two structures: material objects in some sense mirror their locations and regions of spacetime in some sense mirror the objects located at them.\(^1\)

As it turns out, there is a confusing array of logically independent systems of mereological harmony in the literature. I aim to bring some order to these systems. Moreover, I fill in some gaps by offering some natural systems which haven’t been discussed in the literature. The main goal of this paper is to specify the logical relationships between different systems of harmony and to, along the way, explore some of the metaphysical consequences of each system of harmony.

The paper is structured in the following way. First, I make some preliminary remarks about the assumptions I make in the paper. Second, I consider systems of harmony. I begin by considering very weak systems of harmony and then I move on to increasingly stronger systems. For each system of harmony, I briefly mention the most substantive metaphysical consequences of that system. Third, I explore the relationship between systems of harmony and one interesting restriction on the thesis of mereological harmony. I end the paper by clearly laying out the logical relationships of the systems in order that this might serve as a roadmap of possible options for the substantivalist.

\section{Preliminaries}

I begin by making a few remarks regarding the material/spatiotemporal world, mereology, and location. Let’s assume that some form of substantivalism is true. Spacetime regions \emph{really} exist;\(^2\) they are not reducible to

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\begin{enumerate}
\item[I] I take this label from Uzquiano (2011) and Schaffer (2009).
\item[II] Or, perhaps, \emph{spatial} regions really exist. While I’ll proceed by assuming that eternalism is true, everything I say about spacetime regions can be interpreted in a presentist-friendly way.
\end{enumerate}
(or mere constructs from) the spatiotemporal relations holding among material objects, as the relationist wants to maintain. There are two prominent forms of substantivalism in the literature. Let dualism be the view according to which material objects are located at, though not identical to, regions of spacetime. This, I take it, is the historically default substantival position. On the other hand, let supersubstantivalism be the view according to which material objects are identical to their locations. This view is recently defended and explored in Sider (2001), Skow (2005), Schaffer (2009), and Nolan (2014). Since the supersubstantivalist contends that an object is located at a region only if it is identical with that region, a very strong form of harmony seems to naturally fall out of the supersubstantivalist picture of matter and spacetime. While in this paper I don’t assume supersubstantivalism is false, I am more interested in systems of harmony which are available to the dualist. Thus, I will be proceeding as if dualism is the default view. But along the way, I’ll make some remarks which are relevant to the relationship between supersubstantivalism and harmony.

Now for some remarks on mereology. I assume that the dyadic parthood relation which holds among material objects is the same exact relation which holds among regions of spacetime. For simplicity, let’s take parthood as our primitive mereological relation; we’ll let ‘≤’ express this relation. We can then, as usual, define other mereological relations in terms of ‘≤’ and first-order logic. In what follows, I want to be as inclusive as possible and not assume too much concerning the mereology of material objects. Thus I only assume that the parthood relation is a preorder on material objects: it is reflexive and transitive. I do not assume that parthood is anti-symmetric, I do not assume a supplementation principle, nor do I assume an unrestricted fusion schema. The mereology of spacetime regions, on the other hand, is a much less controversial issue. So I will assume that Classical Mereology (CM) is true for regions of spacetime.

3For simplicity, I assume that the parthood relation is a dyadic relation. Those who think that parthood is either a three-place or four-place relation might want to characterize parthood misalignment cases differently. For instance, if parthood is a three-place relation holding among two objects and a time, then a parthood misalignment case might be characterized as x being a part of y at time t without x’s location being a part of y’s location at t, or vice versa. Though there are many interesting ways in which conceptions of parthood as a three- or four-place relation relate to mereological harmony, this exploration would take us too far outside the scope of the current paper.

4There are a number of ways of axiomatizing (CM). Let’s follow Hovda (2008) and say that t is a fusion of a set s in accordance with the following definition:
Lastly, some remarks on location. There are different ways in which an object is located at a region of spacetime. There is some controversy as to which locative relation to take as primitive. Parsons (2007) and Uzquiano (2011), for instance, take weak location as primitive, whereas Casati and Varzi (1999) and Saucedo (2011) take exact location as primitive. There is even some controversy as to what is meant by exact location. Let’s say that, roughly, $x$ is exactly located at $y$ iff $x$ is located at the spacetime region $y$ which has the exact size and shape as $x$, and stands in all the same spatiotemporal relations to other objects as does $x$. For the purposes of this paper, it doesn’t matter which locative relation we take as primitive. Henceforth, I will use “exact location” and “location” synonymously. For simplicity in what follows, I make two assumptions. First, I assume the following principle:

Every material object has an exact location. \hspace{1cm} \text{(Exact Location)}

Moreover, I am going to assume that exact location is a function from objects to regions. We’ll let $f: \mathcal{M} \to \mathcal{R}$ be a total function from the set $\mathcal{M}$ of material objects to the set $\mathcal{R}$ of regions of spacetime; for any object $x$, say that $fx$ is the exact location of $x$. This is what Hovda calls a type-2 fusion. (CM) can now be axiomatized in the following way:

\begin{align*}
F(u, s) &= \forall x(x \in s \to x \leq t) \land \forall y(y \leq t \to \exists x(x \in s \land y \circ x)) \\
\end{align*}

$Fu(t, s) =_{df} \forall x(x \in s \to x \leq t) \land \forall y(y \leq t \to \exists x(x \in s \land y \circ x))$

This is what Hovda calls a type-2 fusion. (CM) can now be axiomatized in the following way:

(1) $(x \leq y \land y \leq z) \to x \leq z$
(2) $\exists x \varphi_x \to \exists ! z Fu(z, \varphi_x)$


6As Parsons (2007) notes, an object $o$ is weakly located at a region $r$ iff $r$ is not completely free of $o$.


8And hence deny that locations are the sorts of things which are located at themselves. Thus, we deny Casati and Varzi (1999)’s principle of Conditional Reflexivity, the principle which says that: if $x$ is located at $y$, then $y$ is located at $y$.

9Even though I make these assumptions for simplicity of presentation, I want to acknowledge that there are a number of reasons some might find the assumptions worrisome.
2 Systems of Mereological Harmony

This section is devoted to exploring a number of systems of mereological harmony. This exploration begins from a simple, but perhaps surprising, observation, originally made in Uzquiano (2011). We can take a mereological relation like parthood and define a number of other relations in terms of parthood and first-order logic.\(^{10}\) And, as it is well known, we can take a number of other mereological relations as primitive (say, overlap, proper parthood, fusion) and define the other mereological relations in terms of our chosen primitive and first-order logic. Suppose we take parthood as our mereological primitive. We can ban what I called parthood misalignment cases by proposing the following principle of mereological harmony:

\[ x \leq y \iff f x \leq f y \]

Parts

However, as Uzquiano (2011) notes, though we can define other mereological relations in terms of parthood, natural analogues of Parts aren’t entailed by Parts. Let’s focus on three cases: overlap, fusion, and proper parthood. Let’s say that \( x \) and \( y \) overlap just in case they have a part in common. Say that \( x \) is a fusion of the \( \varphi \)-things just in case each of the \( \varphi \)-things is a part of \( x \) and for any part of the fusion \( x \), it overlaps at least one of the \( \varphi \)-things. People typically say that \( x \) is a proper part of \( y \) just in case \( x \) is a part of \( y \) and \( x \) is distinct from \( y \), but recall that we aren’t assuming that parthood is anti-symmetric. Therefore, let’s follow an idea in Uzquiano (2011) and say

First, one might think that multilocation is possible. Second, one might think that some objects fail to have exact locations. Suppose there are weird hybrid objects like the fusion of my left hand and \( \pi \). Where is this object located? Though some might claim that it is located exactly where my left hand is located, others might claim that it is only weakly located there, and it isn’t exactly located anywhere. There are plenty of similar cases. For instance, suppose that space is gunky and that there is a point-sized object \( o \). While \( o \) might be weakly located in a number of regions, it might not have an exact location. Or suppose that space is knuggy, i.e., every region has a proper superregion, and that an object \( t \) completely fills in every region. While \( t \) is weakly located everywhere, it doesn’t seem to have an exact location. A total function \( f \) requires that the fusion “my left hand + \( \pi \)”, \( o \), and \( t \) all have exact locations. I take these examples from Parsons (2007). Another case noted in Parsons (2007) and Nolan (2006), which was first shown by Shieva Kleinschmidt to have consequences for the thesis that everything with a weak location has an exact location, is the case of stoic gunk, though see Leonard (2014) for why this case is a bit more complicated than the three examples mentioned above.

\(^{10}\)See Simons (1987) on classical mereology.
that \( x \) is a proper proper part of \( y \) just in case \( x \) is a part of \( y \) and \( y \) has a part which is disjoint from \( x \).\(^{11}\)

Now, even though we can define the above relations in terms of parthood, the following are not entailed by Parts:

\[
\begin{align*}
x \circ y & \leftrightarrow f(x) \circ f(y) \quad \text{Overlap} \\
x Fu\{y : \varphi(y)\} & \leftrightarrow f(x) Fu\{f(y) : \varphi(y)\} \quad \text{Fusions} \\
x < y & \leftrightarrow f(x) < f(y) \quad \text{Proper Proper Parts}
\end{align*}
\]

Overlap says that two things overlap iff their locations overlap, Fusions says that \( x \) is a fusion of the \( \varphi \)-things iff \( x \)'s location is a fusion of locations of the \( \varphi \)-things, and Proper Proper Parts says that \( x \) is a proper proper part of \( y \) iff \( x \)'s location is a proper proper part of \( y \)'s location.

Parts does in fact entail the left-to-right direction of Overlap, given our principle of Exact Location. Suppose \( x \) and \( y \) overlap, and thus share a common part \( z \). By Exact Location, \( z \) has a location. Since \( z \) is a part of \( x \), by Parts, we get that \( fz \) is a part of \( fx \). And likewise, by Parts, we get that \( fz \) is a part of \( fy \). But since \( fz \) is a part of both \( fx \) and \( fy \), it follows by the definition of overlap that \( fx \) overlaps \( fy \).

However, there are very few other entailments.\(^{12}\) For instance, the right-to-left direction is not entailed by Parts. \( fx \) and \( fy \) might very well overlap without \( x \) and \( y \) overlapping. There might be a \( z \) which is a part of both \( fx \) and \( fy \) without anything being located at \( z \). So Parts doesn't entail Overlap. Conversely, Overlap does not entail Parts. Moreover, Parts does not entail Proper Proper Parts and Proper Proper Parts does not entail Parts. Furthermore, Parts does not entail Fusions. However, if one takes fusion as primitive and defines parthood in the usual way, then Fusions does in fact entail Parts.\(^{13}\) However, Fusions entails neither Overlap nor Proper Proper Parts. In sum: even though we can take parthood as primitive in order to define the other mereological relations, none of the analogous principles of mereological harmony are entailed by Parts.

There are many interesting principles of mereological harmony and, as it turns out, very few are entailed by Parts. We are left with a confusing array of systems of mereological harmony. In this section, I attempt to bring

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\(^{11}\)As Uzquiano notes, every proper part is a proper proper part in (CM).

\(^{12}\)Uzquiano (2011) includes a number of relevant countermodels. See pgs. 204-210.

\(^{13}\)That is: \( x \leq y =_{df} y Fu\{x, y\} \)
some order to these systems. I’ll mention a few systems which appear in the
literature and a few natural systems which don’t appear. I’ll end by provid-
ing a novel characterization of what I take to be the most general form of
mereological harmony. Each of the proposals attempt to capture the thought
that the mereological structure of material objects mirrors and is mirrored by
the mereological structure of those objects’ locations in spacetime, in some
way. I begin by noting some very weak systems of harmony and move on to
increasingly stronger systems. Along the way, I’ll mention some important
metaphysical consequences of the different systems of harmony.

2.1 MH₁ Systems

Let us begin with four very weak systems of harmony, systems based on the
principles discussed above. These systems are different forms of what I’ll call
MH₁:

\[
\text{MH}_1 \leq \quad x_1 \leq x_2 \leftrightarrow f(x_1) \leq f(x_2) \quad \text{Parts}
\]

\[
\text{MH}_1o \quad x_1 \circ x_2 \leftrightarrow f(x_1) \circ f(x_2) \quad \text{Overlap}
\]

\[
\text{MH}_1< \quad x_1 < x_2 \leftrightarrow f(x_1) < f(x_2) \quad \text{Proper Proper Parts}
\]

\[
\text{MH}_{1Fu} \quad xFu\{y : \varphi(y)\} \leftrightarrow f(x)Fu\{f(y) : \varphi(y)\} \quad \text{Fusions}
\]

As I said above, besides MH₁Fu entailing MH₁≤, no other entailments hold
among these systems. These are four rather weak options available to the
dualist. Let me now note a few important consequences of these weak sys-
tems of harmony. The consequences of these weak systems are all (perhaps)
somewhat obvious, so I’ll be brief. First, each of these systems ban certain cases of interpenetration. In one type of interpenetration case, the location of an object \( a \) happens to be a part of the location of another object \( b \) without \( a \) being a part of \( b \). This type of interpenetration case is clearly banned by \( \text{MH}_{1\leq} \) (and hence by the stronger \( \text{MH}_{1Fu} \)). According to another type of interpenetration case, the location of \( a \) overlaps the location of \( b \) without \( a \) overlapping \( b \). This type of interpenetration case is clearly banned by \( \text{MH}_{1o} \).

So let’s say that two objects can interpenetrate just in case they can interpenetrate in both of these kinds of ways. Each of the \( \text{MH}_1 \) systems entail the following principle:

\[
\text{It is impossible for material objects to interpenetrate.}
\]

\((\text{No Interpenetration})\)

Second, these weak systems of harmony ban certain mereological misalignment cases. \( \text{MH}_1 \) has as a consequence \((\text{No PM})\). This system is explicitly adopted in Thomson (1998).\(^{14}\) It also appears as the subregion theory of parthood in Markosian (2014).\(^{15}\) It’s worth noting that philosophers who do not accept all four axioms typically report that they do not want to reject all of the above systems (unless, of course, they are moved by considerations concerning interpenetration). Many philosophers seem to want to at least say that parthood misalignment cases are impossible - with the notable exception being Saucedo (2011), who thinks all mereological misalignment cases are possible. Moreover, it’s worth noting that \( \text{MH}_{1\leq} \), or at least something as strong as \( \text{MH}_{1\leq} \), is necessary for banning such parthood misalignment cases. In particular, \((\text{No PM})\) is not a consequence of \( \text{MH}_{1o} \) or \( \text{MH}_{1<} \). To those dualists who think that parthood misalignment cases are impossible, both \( \text{MH}_{1Fu} \) and \( \text{MH}_{1<} \) are live options.

In addition to parthood misalignment cases, one can consider other mereological misalignment cases. For instance, an overlap misalignment case is a case where two objects overlap while their locations fail to overlap, or vice versa. A proper proper parthood misalignment case is a case where an object is a proper proper part of something while its location fails to be a proper

\(^{14}\)It’s worth noting that she relativizes the parthood relation to times. But all of the systems I consider can also be relativized to times or, say, regions of spacetime for those dualists who reject the notion that parthood is a 2-place relation.

\(^{15}\)The formulation there, however, distinguishes between parthood on regions and a relation of subregionhood.
proper part of the other thing’s location, or vise versa. And a fusion misalignment case is a case where something \( t \) is the fusion of some \( \varphi \)-things without \( t \)'s location being a fusion of the locations of the \( \varphi \)-things, or vice versa. Consider the following theses:

It is impossible for \( x \) to overlap \( y \) without \( x \)'s location overlapping \( y \)'s location, and vice versa. **No Overlap Misalignment (No OM)**

It is impossible for \( x \) to be a proper proper part of \( y \) without \( x \)'s location being a proper proper part of \( y \)'s location, and vice versa. **No Proper Proper Parthood Misalignment (No PPPM)**

It is impossible for \( x \) to be a fusion of some \( \varphi \)-things without \( x \)'s location being a fusion of the locations of the \( \varphi \)-things, or vice versa. **No Fusion Misalignment (No FM)**

**No OM** is a consequence of \( \text{MH}_{1o} \) but not the others, **No PPPM** is a consequence of \( \text{MH}_{1c} \) but not the others, and **No FM** is a consequence of \( \text{MH}_{1Fu} \) but not the others.

### 2.2 MH\(_2\)

While there are plenty of possible systems comprised of two or three of the above mirroring axioms, let’s consider a slightly stronger system in the literature which proposes all four of the axioms above: the system discussed in Uzquiano (2011). Let \( \text{MH}\(_2\) \) be the system comprised of the following axioms and fusion axiom schema:

\[
\text{MH}\(_2\)
\]

\[
x \leq y \leftrightarrow f(x) \leq f(y)
\]

\[
x \circ y \leftrightarrow f(x) \circ f(y)
\]

\[
x < y \leftrightarrow f(x) < f(y)
\]

\[
x Fu\{y : \varphi(y)\} \leftrightarrow f(x) Fu\{f(y) : \varphi(y)\}
\]

Uzquiano’s (2011) \( \text{MH}\(_2\) \) entails that all four misalignment cases mentioned above are impossible. As Uzquiano points out, it’s worth noting that this
system has some surprisingly strong consequences. In particular, the follow-
ing two theses are consequences of $\text{MH}_2$:

It is impossible for material gunk to be located in atomistic space.

\textit{(No Gunk in Pointy Space)}

It is impossible for unextended material atoms to be located in White-
headian (gunky) space.

\textit{(No Unextended Atoms in Gunk)}

An explicit argument against \textit{(No Gunk in Pointy Space)} is provided in Mc-
Daniel (2006). It’s worth stressing that if one wants to ban all four misalign-
ment cases mentioned above, one also needs to ban gunky objects in pointy
space and unextended material atoms in gunky space.

\subsection*{2.3 $\text{MH}_3$}

Let’s next consider an even stronger system of harmony. We can define the
1-place predicates ‘is simple’, ‘is complex’, ‘is gunky’, and ‘has exactly $n$
parts’ in the following straightforward ways:

\begin{align*}
  x \text{ is simple} & \equiv_d \neg \exists y (y < x) \\
  x \text{ is complex} & \equiv_d \exists y (y < x) \\
  x \text{ is gunky} & \equiv_d \forall y (y \leq x \rightarrow \exists z (z < y)) \\
  x \text{ has exactly } n \text{ parts} & \equiv_d \exists y_1 \ldots \exists y_n ((y_1 \neq y_2 \land \ldots \land y_{n-1} \neq y_n) \land (y_1 \leq \\
  & \quad x \land \ldots \land y_n \leq x) \land \forall z (z \leq x \rightarrow z = y_1 \lor \ldots \lor z = y_n))
\end{align*}

Let $\text{MH}_3$ be the system considered in Saucedo (2011).\footnote{But note that Saucedo (2011) just considers the system. He doesn’t endorse it. As mentioned above, he argues that all misalignment cases are possible.} It is the system of
harmony comprised of the following axioms and axiom schemas:

\begin{align*}
  x \leq y & \leftrightarrow f(x) \leq f(y) \quad \text{Parts} \\
  x \circ y & \leftrightarrow f(x) \circ f(y) \quad \text{Overlap} \\
  x < y & \leftrightarrow f(x) < f(y) \quad \text{Proper Proper Parts} \\
  x Fu\{y : \varphi(y)\} & \leftrightarrow f(x) Fu\{f(y) : \varphi(y)\} \quad \text{Fusions}
\end{align*}
\( x \) is simple \( \leftrightarrow f(x) \) is simple \hspace{1cm} \textbf{Simplicity}
\( x \) is complex \( \leftrightarrow f(x) \) is complex \hspace{1cm} \textbf{Complexity}
\( x \) is gunky \( \leftrightarrow f(x) \) is gunky \hspace{1cm} \textbf{Gunkiness}
\( x \) has exactly \( n \) parts \( \leftrightarrow f(x) \) has exactly \( n \) parts \hspace{1cm} \textbf{Number of Parts}

The first four axioms are what Saucedo calls \textit{external alignment axioms} and the latter four are what he calls \textit{internal alignment axioms}. As Saucedo notes, the external and the internal are logically independent from one another. For the sake of time, I won’t provide all the countermodels necessary to demonstrate this.\(^{17}\) I will offer one, however, so the reader can get a feel for how easy it is to see that these two classes of axioms are independent from each other. Consider a model where there is only one material thing \( a_1 \) and \( a_1 \)’s location is gunky.\(^{18}\) In this model, \textit{Parts}, \textit{Overlap}, \textit{Proper Proper Parts}, and \textit{Fusions} are all satisfied, and yet \textit{Gunkiness} is not, since \( fa_1 \) is gunky and \( a_1 \) is not.

There are a number of metaphysical consequences of this system of harmony which are controversial. Perhaps the most controversial is the following thesis:

\[ \text{It is impossible for a simple object to be located at a complex region.} \]
\[ \text{(No Extended Simples)} \]

This case is banned by \textit{Simplicity}.\(^{19}\) So if one is inclined to think that extended simples are possible, one cannot accept \textit{MH}_3.\(^{20}\) It is important to note just how strong \textit{MH}_3 really is. It is very natural to think that while I am composed of finitely many atoms, my location is composed of continuum

\(^{17}\)It’s also worth noting that this system is slightly redundant. For example, \textit{Simplicity} is equivalent to \textit{Complexity}.

\(^{18}\)In other words, the following model \( \mathcal{M} = \{D,I\} \):

\[
\begin{align*}
D &= \{ a_1, r_1, r_2, r_3, \ldots \} \\
I(\leq) &= \{ \langle a, a \rangle, \ldots, \langle r_2, r_1 \rangle, \langle r_3, r_1 \rangle, \langle r_4, r_2 \rangle, \langle r_5, r_2 \rangle, \langle r_6, r_3 \rangle, \langle r_7, r_3 \rangle, \ldots \} \\
I(f) &= \{ \langle a_1, r_1 \rangle \}
\end{align*}
\]

\(^{19}\)For defenses of the possibility of extended simples, see Markosian (1998), Parsons (2000), and McDaniel (2007).

\(^{20}\)Of course, this relies on the assumption that spacetime atoms are themselves unextended entities. See Braddon-Mitchell and Miller (2006) for a defense of why this might not be the case. Thanks to an anonymous referee for pointing this out.
many atoms. However, MH$_3$ has the following consequence:

It is impossible for an object to have a different number of parts than its location. (No Numerical Difference)

This is quite a strong requirement to place on the relationship between material objects and their locations.

### 2.4 Full-Blown Harmony

MH$_3$ contains a number of mirroring axioms. But it is natural to wonder what a perfectly general theory of harmony would look like. This section is devoted to exploring attempts to unify systems like MH$_2$ and MH$_3$. Say that full-blown harmony is the view that there is an isomorphism between the set of material objects and the set of regions of spacetime. Below I consider two systems of full-blown harmony: MH$_4$ and the slightly stronger MH$_4^+$.

Let ‘$\varphi(x_1, ..., x_n)$’ be any mereological sentence with $n$ free variables, where $\alpha$ is a mereological sentence iff $\alpha$ is a sentence of first-order logic (without identity), all of whose non-logical vocabulary is defined in terms of ‘$\leq$’ and logic (more precisely: in terms of negation, conjunction, and the existential quantifier). Say that MH$_4$ is the system of mereological harmony comprised of the following mirroring schema:

$$\varphi(x_1, ..., x_n) \leftrightarrow \varphi(fx_1, ..., fx_n)$$

Complete Mirroring

There is an argument to be made in favor of full-blown harmony. I call it the argument from elegance. Suppose a dualist accepts something like MH$_3$, or perhaps even MH$_2$. It is natural to ask what it is that unifies these systems of harmony. In some sense, these moderate views of harmony seem to be nothing more than a disunified scattering of mirroring principles. Full-blown

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21It’s important to note that only mereological sentences are substitutable. For instance, predicates like ‘is a material object’ or ‘is a region’ are not substitutable for ‘$\varphi$’ because they are not expressible in mereology. Otherwise, they’d clearly hold of some material objects, or regions, though not vice versa. Also, names are banned. So when ‘$r$’ is a name, ‘$r \leq x$’ is not substitutable for ‘$\varphi x$’. Thanks to Ted Sider for noting this point about names.
harmony, on the other hand, is a perfectly unifying and elegant theory of harmony. And this, I think, is a reason to take it seriously.

However, MH$_4$ has a number of consequences and so there are plenty of reasons to think that it is simply way too strong. Not only does full-blown harmony entail all of the preceding consequences, but it entails even stronger metaphysical consequences. For instance, consider the expansions problem. Say that an expansion of an object $o$ is an object $o'$ which is located at a proper superregion of $o$'s location. For example: though my body isn’t an expansion of my body, my body is an expansion of my arm. Let’s say that a final expansion is an object $e$ which is both an expansion of some distinct object $e'$ and an object which itself has no expansions. MH$_4$ has as a consequence the following thesis:

If there is empty space, it is impossible for there to be a final expansion.

(No Final Expansions)

For any object $o$, since $fo$ is a proper part of something, it follows from Complete Mirroring that $o$ is a proper part of something. And since my location is a proper part of $n$ many things, it follows that I too am a proper part of $n$ many things.

Any dualist who holds to some moderate answer to the Special Composition Question is not going to appreciate this consequence. However, there’s even a worry for the universalist. Suppose that $m$ is the fusion of all material objects. Assuming that the dualist doesn’t believe in a material plenum (i.e., doesn’t believe that every region of space is occupied by a material object), $m$’s location is surely a proper part of something, and hence, so too is $m$. But since $m$ is the fusion of every material object, there isn’t a material object of which it can be a proper part. Hence, it must be a proper part of something else. The universalist is left with two options. First, $m$ might be a proper part of a proper superregion of $fm$. But this is typically anathema to dualist substantivalism. On this view, material objects are not parts of regions of spacetime; material objects are located at regions of spacetime. Second, perhaps the universalist could say that $fm$ is a part of a cross-categorial fusion of $m$ and some superregion of $fm$. While this is a fairly nonstandard thing

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22 One wrinkle concerns (Number of Parts). Since the identity relation appears in the definiens of (has exactly $n$ parts), (Number of Parts) isn’t an instance of MH$_4$. Nonetheless, (No Numerical Difference) is guaranteed by MH$_4$, as objects with different numbers of parts can be distinguished purely in terms of mereology without identity.
for the dualist to say, this certainly is an option available to the dualist.

I now want to offer a variant of MH$_4^-$. Let’s say that MH$_4^+$ is the system which is just like MH$_4^-$ except that we allow as substitutable in Complete Mirroring any mereological sentence $\varphi(x_1, ..., x_n)$ all of whose non-logical vocabulary is defined in terms of ‘$\leq$’, conjunction, negation, the existential quantifier, as well as ‘$=$’. MH$_4^+$ is slightly stronger than MH$_4^-$. Interestingly enough, MH$_4^+$ has as a consequence the following thesis:

$$\text{It is impossible for } x \text{ and } y \text{ to be distinct and co-located.}$$

((No Co-location))

MH$_4^+$ entails that the statue and the lump cannot be distinct co-located objects. The following instance of Complete Mirroring bans the possibility of co-location:

$$x = y \leftrightarrow fx = fy$$

Identity

Suppose the statue $s$ and the lump $l$ are both distinct and located at some region $r$. Since $fs = fl$, this principle entails that the statue is identical to the lump, contrary to hypothesis.$^{23}$

2.5 Half-Harmony

Before moving on, I want to very briefly mention two other classes of systems. This paper is primarily concerned with systems of harmony, i.e., systems representing the thought that mirroring occurs in both directions. There are, however, half-harmony views. For the sake of time, I will simply mention the strongest system in each of these two classes. According to the first class of

$^{23}$Thanks to Aldo Antonelli for first bringing to my attention Identity. I should make two brief remarks concerning (No Co-location). First, in the setup of the paper, we noted that we weren’t going to take locations to be located at themselves. If we had, then we would have run into some additional trouble with MH$_4^+$. In particular, this system of harmony is inconsistent with (the dualist idea) that my body and my location are co-located and yet distinct. Second, one of the responses to the expansions problem for the universalist was to admit of cross-categorial fusions. It’s worth noting that if one takes this route, and one also thinks that such entities are located entities, then MH$_4^+$ causes some trouble. In particular, my body and the fusion of my body and my location are co-located, yet distinct. One, however, could naturally say that cross-categorial fusions of this sort simply lack exact locations.
systems, the mereological structure of material objects in some way is determined by the mereological structure of their locations (though not necessarily vice versa). This thought can be captured by the following principle:

\[ \varphi(f_x_1, \ldots, f_x_n) \rightarrow \varphi(x_1, \ldots, x_n) \quad \text{Object Mirroring} \]

Something like this view seems to be defended recently in Markosian (2014)'s spatial approach to mereology. On the other hand, some might think that the mereological structure of material objects determines the mereological structure of those objects' locations (though not necessarily vice versa). This thought can be captured by the following principle:

\[ \varphi(x_1, \ldots, x_n) \rightarrow \varphi(f_x_1, \ldots, f_x_n) \quad \text{Region Mirroring} \]

Of course, there are half-harmony versions of MH\textsubscript{3}, MH\textsubscript{2}, and each version of MH\textsubscript{1}. I won’t take the time to explore the complex relationship among the members of these classes, but they too are mostly logically independent from one another.

### 3 Host-Harmony

With the exception of MH\textsubscript{1} ≤, all of the above systems of harmony have an interesting feature. The definitions of overlap, proper proper parthood, fusion, gunkiness, simplicity, complexity, and number-of-parts contained in the different systems all contain quantifiers in the \textit{definiens}. Moreover, the quantifiers are completely unrestricted. In particular, the quantifiers over regions of spacetime range over the entirety of spacetime. Even though Parts is weaker than full-blown harmony, it turns out that with a certain restriction on our quantifiers, Parts is equivalent with an interesting version of full-blown harmony.

\[ \text{24} \text{It is worth noting, however, that Markosian (2014) also seems to accept something like MH}_{1} \leq \text{ when he proposes his } \textit{subregion theory of parthood. So it looks like the view in Markosian (2014) is a conjunction of MH}_{1} \leq \text{ and Object Mirroring, though none of this is explicitly spelled out in the paper.} \]

\[ \text{25} \text{This is assumed, at least with respect to parthood, in Jansen and Schulz (2014), though the location relation they use is, as they admit, slightly broader than the location relation typically discussed and the notion I assume in this paper.} \]

\[ \text{26} \text{Thanks to Gabriel Uzquiano for originally pointing this out to me.} \]
In what follows, when we quantify over entities with locations (on the left-sides of the mirroring biconditionals), we explicitly restrict our quantifiers to range over material objects. This is, however, nothing new: this is implicitly what we were doing in the above systems. Now for the novel idea. Say that a host is a region at which an object is exactly located. In what follows, when we quantify over regions of spacetime (on the right-sides of the mirroring biconditionals), we restrict our quantifiers to only range over regions which are hosts.

3.1 HH−

For any formula $\varphi(x_1, \ldots, x_n)$, let $\varphi(x_1, \ldots, x_n)^M$: restrict any quantifier appearing in $\varphi$ to only range over material objects. And let $\varphi(fx_1, \ldots, fx_n)^H$: restrict any quantifier appearing in $\varphi$ to only range over regions which are hosts. Now consider the host-version of Overlap:

$$(x \circ y)^M \leftrightarrow (fx \circ fy)^H$$

Host-Overlap

or,

$$\exists z(z \leq x \land z \leq y)^M \leftrightarrow \exists z(z \leq fx \land z \leq fy)^H$$

Host-Overlap

Host-Overlap is weaker than Overlap. Overlap says that two objects have a part in common iff their locations have a part in common (where any region of spacetime will do). Host-Overlap says that two objects have a material object as a part in common iff their locations have a host-region in common. Consider an example. Suppose two extended simples $a$ and $b$ interpenetrated. Suppose half of $a$ and half of $b$, roughly, come to share the same region of space. While this is ruled as impossible by Overlap, this is perfectly consistent with Host-Overlap. Since no host is a common part of $fa$ and $fb$, we aren’t forced to say that $a$ and $b$ overlap. Notice that once we make this restriction, Parts does in fact entail Host-Overlap. Suppose $a$ and $b$ overlap. It follows that they have a part in common, $c$. By Parts, $fc$ is a part of both $fa$ and $fb$, and so $fa$ and $fb$ overlap. Now, for the other direction, suppose $fa$ and $fb$ host-overlap, by which we mean that they have a host in common $fc$, for some $c$. By Parts, $c$ is a part of both $a$ and $b$, and hence, $a$ and $b$ overlap. So Host-Overlap follows from Parts.

Whereas full-blown harmony is the view that there is an isomorphism
between the set of material objects and the set of regions of spacetime, let’s say that *host-harmony* is the view that there is an isomorphism between the set of material objects and the set of hosts. Just as we had two versions of full-blown harmony, we have two versions of host-harmony. Let’s say that \( \text{HH}^− \) is the view committed to every instance of the following schema:

\[
\text{HH}^− \\
\varphi(x_1, ..., x_n)^M \leftrightarrow \varphi(fx_1, ..., fx_n)^H
\]

where any mereological sentence with \( n \) free variables defined in terms of ‘\( \leq \)’ and logic (without identity) can be substituted for \( \varphi \).

In Section 2, we noted that *Parts* entails neither direction of *Proper Proper Parts*. Nor does it entail *Fusions, Gunkiness, Simplicity*, or any of the other axioms mentioned above. Interestingly enough, once we build the quantifier restriction in to the different formulas, *Parts* entails the host-versions of each of these. Moreover, it turns out that \( \text{MH}_{1\leq} \) is equivalent to \( \text{HH}^− \). In other words, the thesis that \( x \) is a part of \( y \) iff \( x \)’s location is a part of \( y \)’s location is equivalent to the thesis that there is an isomorphism between the set of material objects and the set of hosts. Obviously, \( \text{HH}^− \) entails \( \text{MH}_{1\leq} \). We can show that \( \text{MH}_{1\leq} \) entails \( \text{HH}^− \) by induction on the complexity of formulas. I’ll show the base case and the quantifier case (the cases for conjunction and negation are trivial). For the base case, consider a sentence of the form: \( a \leq b \). By *Parts*, “\( a \leq b \)” is true iff “\( fa \leq fb \)” is true. So “\( (a \leq b)^M \)” is true iff “\( (fa \leq fb)^H \)” is true. Now for the quantifier case. Consider a sentence of the form: \( \exists y A(y) \). For inductive hypothesis, assume that “\( A(a, b)^M \)” is true iff “\( \exists x A(x, b)^M \)” is true. Suppose that “\( \exists x A(x, b)^M \)” is true.
Suppose $a$ is an extended simple located at $r$. $a$ doesn’t have a proper part, but $r$ doesn’t have a proper-part which is a host. Let’s call these host-parts. $r$ by definition has proper parts; but it doesn’t have any proper host-parts. So extended simples aren’t ruled out by this instance of Complete Host-Mirroring.

Second, host-harmony doesn’t entail (No Gunk in Pointy Space). Consider the following instance of Complete Host-Mirroring (the analogue to the instance of full-blown harmony which did ban gunky objects in pointy space):

$$\forall z (z \leq x \rightarrow \exists w (w < z))^M \leftrightarrow \forall z (z \leq fx \rightarrow \exists w (w < z))^H$$

Or, in English: all of $x$’s parts have proper proper parts iff all of $fx$’s host-parts have proper proper host-parts. The unrestricted condition ‘all of $fx$’s parts have proper proper parts’ is surely unsatisfied, since $fx$ is composed of points. But the condition above, that all of $fx$’s host-parts have proper proper host-parts, can in fact be satisfied. Imagine a gunky object $o$, located at $r$, where all of $o$’s proper proper parts are located at sub-regions of $r$; $r$ is host-gunky, even though it’s ultimately composed of points. So host-harmony permits gunky objects to be located in pointy space.

Third, host-harmony doesn’t entail (No Ultimate Expansions). Consider the host-version of the instance of full-blown harmony which did entail the thesis:

$$\exists y (x < y)^M \leftrightarrow \exists y (fx < y)^H$$

Suppose one has some moderate answer to the Special Composition Question and suppose $o$ is an object which is not a proper proper part of anything. Since $o$’s location isn’t a proper proper part of any host, it doesn’t follow that $o$ is a proper proper part of anything. Now suppose one is a universalist, and say that $m$ is the fusion of every material object. The location of $m$ isn’t a proper proper part of any host, and so the universalist isn’t forced to say anything about what $m$ is a proper proper part of: perhaps it’s not a proper

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27See Shiver (2015) for a discussion on how to formulate the claim that “everything is ultimately composed of atoms.”
proper part of anything.

It’s also important to note that the host-version of the expansions generating axiom is compatible which a recent supersubstantivalist position defended in Nolan (2014), whereas the unrestricted axiom is not. Nolan claims that while all material objects are identical to regions of spacetime, not all regions are material objects. Moreover, Nolan defines a relation of material parthood, and wants to claim that material objects are only parts of other material objects. So, on this supersubstantivalist picture, the fusion $m$ of every material object is not a proper proper material part of anything. Nolan’s supersubstantivalism meshes nicely with host-harmony, but not full-blown harmony.\(^{28}\)

So there are a number of consequences of full-blown harmony which are not inherited by host-harmony. However, the view isn’t completely trivial. There is an interesting consequence of host-harmony (and hence, $\text{MH}_{1\leq}$). Say that a crowded simple is a complex object located at a simple region of spacetime. Host-harmony entails the following thesis:

It is impossible for there to be a crowded simple. \((\text{No Crowded Simples})\)

This follows from the following instance of Complete Host-Harmony:

\[
\exists z(z \leq x \land x \not\leq z)^{\mathcal{M}} \leftrightarrow \exists z(z \leq fx \land fx \not\leq z)^{\mathcal{H}}
\]

Since a complex object has a part of which it is not a part, its location must have a host-part of which it is not a host-part. One plausible example of a crowded simple is the fusion of two co-located bosons (assuming bosons are simple); it is interesting to note that a system as weak as $\text{MH}_{1\leq}$ bans such cases.

Though this paper has been primarily concerned with views of harmony, as opposed to views of half-harmony, it is worth noting that the ban on crowded simples is entailed merely by the left-to-right direction of $\text{MH}_{1\leq}$.\(^{29}\)

\(^{28}\)In the preliminary section of this paper, I assumed that the parthood relation on material objects is the same parthood relation on regions of spacetime. This is something which the view Nolan develops rejects; however, a host-version of the expansions generating instance of harmony with the caveat that parthood is a relation on material objects and subregionhood is a relation on regions, is compatible with Nolanian supersubstantivalism; and the unrestricted full-blown instance is not.

\(^{29}\)Though we didn’t show that the left-to-right direction of $\text{MH}_{1\leq}$ is equivalent to the
So the only direction of Parts needed to ban crowded simples is the (completely uncontroversial) left-to-right direction, the direction assuring us that $x$ being a part of $y$ is a sufficient condition for $fx$ being a part of $fy$.

So even though $\text{MH}_{1\leq}$ initially seemed quite tame, it turns out that it has a little more of a bite to it. According to even this very weak system of harmony, there can be no crowded simples.

### 3.2 HH$^+$

Lastly, I briefly mention one version of host-harmony which is slightly stronger than HH$. Let HH$^+$ be just like HH$^-$ in being committed to every instance of Complete Host-Mirroring, except that in this system we allow mereological sentences defined with identity to count as substitutable for $\varphi$.

In addition to inheriting the consequences of HH$, HH$^+$ entails (No Co-location). This follows from HH$^+$ for the same reason it follows from $\text{MH}_{4+}$; namely, it follows from:

$$x = y \iff fx = fy$$

Identity

And it’s worth noting that not only is $\text{MH}_{1\leq}$ equivalent to HH$, but $\text{MH}_{1\leq}$ together with (No Co-location) is equivalent with HH$^+$. We can easily check that $\text{MH}_{1\leq}$ together with (No Co-location) entails HH$^+$, again, by induction on the complexity of formulas. The proof is just like the proof for the claim that $\text{MH}_{1\leq}$ entails HH$^-$ except that we must consider one more base case; consider a sentence of the form: $a = b$. We need to show that “$(a = b)^{\mathcal{M}}$” is true iff “$(fa = fb)^{\mathcal{H}}$” is true. The left-to-right direction follows by the fact that location is a function and the right-to-left direction follows from (No Co-location).

### 4 A Concluding Roadmap

As we’ve seen, there is a confusing array of systems of harmony available to the substantivalist. The goal of this paper has been to bring some order to this confusing relationship of systems. Along the way, we’ve explored some of the more interesting metaphysical consequences of the different systems of left-to-right direction of HH$, this proof can be read off of the inductive proof above for the thesis that $\text{MH}_{1\leq}$ entails HH$^-$.
harmony. And we’ve seen that many of the natural systems of harmony fail to make a distinction between quantification over the entirety of spacetime and quantification merely over hosts. We’ve seen that restricting our quantifiers to only range over hosts suggests a couple of formulations of harmony, one of which turns out to be equivalent to one of the weakest systems of harmony on the market.

I end the paper with a diagram representing a roadmap for substantivalists. The lattice below is intended to report two important things. First, it describes the logical relationships between each of the systems of mereological harmony. Systems represented by nodes connecting to higher nodes are entailed by the system represented by the higher node. Second, the lattice contains a list of the most important consequences for each system of harmony. Each system of harmony inherits all of the consequences from systems represented by connected lower nodes, though not vice versa. So we end the paper by providing the following roadmap of possible views for the substantivalist who is tempted to accept some form of mereological harmony.

SYSTEMS OF HARMONY AND THEIR CONSEQUENCES

\footnote{For instance, though $\text{MH}_4^+$ has every principle in parentheses as a consequence, $\text{MH}_{1^o}$ only has the two consequences written underneath in.}

\footnote{For helpful comments and discussion on earlier drafts of this paper (some drafts being quite distant in the past), I’d like to thank Aldo Antonelli, Andrew Bacon, Mark Bala- guer, Jacek Brzozowski, Tim Crane, Scott Dixon, Maegan Fairchild, Kit Fine, Thomas Hall, Daniel Korman, David Kovacs, Bernard Molyneux, Josh Parsons, Ben Rhors, Raul Saucedo, Jonathan Schaffer, Adam Sennet, Ted Shear, Anthony Shiver, Ted Sider, Robert Stalnaker, Michael Tooley, Jim Van Cleve, Emanuel Viebahn, Andy Yu, Alexander Zambrano, audiences at Oxford University, Yale University, Stanford University, Colorado (Boulder), UC Davis, Western Washington University, the University of Cologne, the University of Belgrade, and the 2013 Pacific APA. Special thanks to two anonymous referees, Cody Gilmore, John Hawthorne, Shieva Kleinschmidt, Jeff Russell and Gabriel Uzquiano for extensive comments on multiple drafts of the paper.}
References


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